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Cost-based pricing model with value-added tax and corporate income tax for a supply chain network

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ABSTRACT

This article aims to propose the short-term cost-based pricing method of supply chain network with the consideration of value-added tax (VAT) and corporate income tax. First, the average cost function of each business unit in supply chain network is given, and the average cost function is taken as the monotone mapping in n-dimensional space. According to Kantorovich theorem, the existence and uniqueness of equilibrium point where the cost equals the income is discussed. When the demand function satisfies certain conditions, there generally exist many equilibrium points for cost-based pricing. Moreover, the iteration method for finding one of the equilibrium solutions is given. Then, tax burden of producers and consumers is described and illustrated with an example.

1. Introduction

Supply chain is the basic organizational structure of economic activities such as production and sales. Supply chain network can be defined as follows: the entire process in which raw material supply, production and processing, product transportation, inventory, sales of end products, as well as after-sales service provided to users, are linked.

The supply chain network usually consists of suppliers, manufacturers, warehouses, distribution centers, and dealers and so forth. Suppose the supply chain network consists of n production and business units which are denoted by n nodes correspondingly. Each production and business activity requires the investment of raw materials, fixed capital and labor hours. For instance, if a specific node represents the production activities of a certain product, the purchase of raw materials from outside the system is needed, or, the products are purchased from other nodes as the raw material input of this node; additionally, plants and equipment (fixed capital) as well as the input of labor hours are also required. If a specific node denotes the transportation activity and a certain product is transported from A (A node) to point B (B node), then the product in A place is seen as an input of transportation activities, while after its transportation to point B, the product can be considered as the output of transportation activities. Transportation vehicles, etc. can be seen as fixed capital investments, while transportation staffs are seen as the input of labor hours. So far, there have already been a great deal of literatures on the operation and management of supply chain network (for instances see [1–6]). The analysis of the production, pricing and profits at each node of the supply chain network is usually conducted according to the following steps: first, each node is treated as an independent business unit. The output at the *ith* node is q_i, which can not only represent the product quantity of a certain product produced by a specific production unit, but also the ton-kilometers of transportation offered by the transportation unit. The total cost TC_i of the *ith* node consists of the fixed cost C_i and variable cost $VC_i(q_i)$, e.g.

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$$TC_i(q_i) = C_i + VC_i(q_i), \tag{1}$$

where the fixed cost C_i is a constant, the value of variable cost is related to the output q_i. The maximization of profits π_i at each node can be described by the following equation:

$$Max \quad \pi_i = p_i q_i - TC_i(q_i). \tag{2}$$

The profits are maximized at each node in order to reach the Cournot equilibrium and to obtain the corresponding price, output and profits [7].

The above method has been adopted in many research works (see [3,8,9] for instances). However, a number of key problems remain to be solved, and further improvement is needed:

(I) How to determine the value of fixed cost C_i? Eq. (1) suggests that the fixed cost C_i is a constant, but how can it be determined according to the actual data?

In the actual production and business activities, each node requires input of fixed capital and labor hours. For instance, regardless of whether the production at the *ith* node has been started or not, the processing equipments costing K_{1i} dollar, the plant costing K_{2i} dollar, the transportation vehicles costing K_{3i} dollar and the fixed labor hours of L_i per month (suppose that the production cycle is 1 month) are needed. Then, the value volume of fixed cost measured in monetary unit is as follows:

$$C_i = (r + \delta_1)K_{1i} + (r + \delta_2)K_{2i} + (r + \delta_3)K_{3i} + wL_i, \tag{3}$$

where $\delta_1, \delta_2, \delta_3$ denotes different types of fixed capital depreciation rate of this month, respectively; r denotes net return rate of fixed capital of this month; w denotes the monthly wage rate of fixed labor hours; data about the fixed capital K_{ji} and labor hour L_i can be collected in practice; then, the fixed cost C_i can be calculated according to Eq. (3). The variation of r and w can affect the value of fixed cost C_i.

(II) The public fixed capital and public service are not considered in the supply chain network above.

In practice, public infrastructures are required at each node of supply chain network, as well as the public service provided by public management sectors. Therefore, each node or management unit should pay tax as the return of public fixed capital and public service. At present, the value-added tax (VAT) and corporation income tax is imposed on enterprises and business units in China, and the tax collection affects the pricing and output quantity of each business unit.

Besides, Cournot equilibrium solution of supply chain network is generally discussed in literatures. Cournot equilibrium price is higher than the cost price, but the cost price is more reasonable, for instance see [10,11]. There are few research works on the cost-based pricing in supply chain network currently that may be nevertheless quite useful in practice. This is due to the fact that Cournot competition usually refers to output competition, e.g. each business unit calculates the output corresponding to the maximized profit in order to reach Cournot equilibrium. In actual business activities, however, the competition among various business units is usually the price competition, with each business unit determines the most favorable price according to the market. When the income of each business unit equals the cost pricing, the net return rate r_i of fixed capital and the wage rate w_i of labor hour vary for each unit. If these two rates are identical among different units, then it can be said that the entire network has reached reasonable equilibrium.

The cost-based pricing model can be applied to cost control and management in an enterprise. The production, transportation, inventory, sales and other activities of an economic entity with independent accounting (such as company, enterprise) can be described using the supply chain network model. Such an economic entity always seeks to reduce costs and increase profits. The cost-based pricing model can be used to understand which links are more sensitive to the cost of products, so as to implement management and control over the production cost. Delicacy management of the production process in an enterprise also involves product quality management apart from cost management. Only cost calculation but no quality problem is considered in this paper, that is to say, each product has only one level of quality. Castillo-Villar, K. K, Smith, N. R, & Simonton, J. L and others have made extensive researches on the cost of quality of the supply chain by establishing models (see [12–15]). On the basis of the research results above, further study can be conducted over the quality of the production process and cost management in order to benefit the modern management of an economic entity with independent accounting.

This study suggests a method of cost-based pricing of supply chain network when VAT and the corporate income tax are taken into account. Then the following conditions should be met by each business unit in this type of supply chain network: the fixed capital invested during the production process is the given constant that cannot be adjusted in a short term; thus, it is only applicable to the description of production and business activities in the short term. With the increase in output, the average cost decreases gradually. For ordinary production units, there always exists the maximum production capability. Within this capability, the above method is applicable. Especially for the production of information products with lower reproduction cost, the production capability is relatively greater, and the average cost decreases with the increase of output (see [16]). The situation of overloading in production is not included in this study. The approach suggested in this study has the following characteristics:

(1) When VAT and corporate income tax are taken into account, this cost-based pricing method is much closer to real situations. Fluctuation of VAT rate, which has huge influence on corporate production, is linked to production cost and profits. If VAT rate is too high, there may be no solution to cost-based pricing model, which means that the enterprise encounters losses. The change of VAT rate is normally decided by the government. Enterprises can only make decisions over the specified VAT rate. But the method proposed in this paper can be used for cost control over the production process. For example, when changes occur to the production process (the relational graph of production units) or equipment, personnel, or raw materials, the cost will fluctuate. Meanwhile, the calculation can help understand which factors are more sensitive to production cost and the changes of which factors can greatly reduce the production cost.

Production and operation activities in real life can usually be described with the supply chain, and the cost-based pricing model has extensive applications. Besides its role in cost control over the established production process, the method presented in this paper can also be used in cost estimation of uncompleted production process. For example, if one contractor undertakes an engineering project, the project can be described using the supply chain. Thus the cost price of such project will be calculated or estimated.

(2) Data collection is relatively easier, e.g. the data about the raw material, fixed capital, labor hours are easy to collect.
(3) Generally, there exist many equilibrium solutions to cost-based pricing. The first one is the lowest equilibrium price solution. This is the most important solution, since it reflects the fair pricing of products, where the stable balanced point may be found out by the iteration method and iteration algorithm can be used for easier calculation. There may exist the highest equilibrium price solution. It is the unstable balanced point, it cannot be found directly according to iteration method. The prices obtained with other equilibrium price solutions vary, ranging from the highest to the lowest. When the prices are within the region encircled by various equilibrium points, the excess profits higher than the costs are obtained.

2. The cost-based pricing model of supply chain network with VAT and corporate income tax considered

Suppose that the supply chain network contains n production and business units that are represented by n nodes correspondingly. Each unit produces one product. The parameters of the *ith* unit are as follows:

Sets:

$i, j \in (1, \ldots, n)$, the supply chain network contains n production units or nodes, such as *i, j*.

Variables:

K_i— value volume of invested fixed capital measured in monetary unit is a constant. For instance, it can be the gross value of the plants and equipments used in this unit. The fixed capital can also be divided into several types according to Eq. (3). However, only one type of fixed capital is assumed to be involved below, and the calculation method is the same with the case in Eq. (3).

L_i— the invested labor hours, a constant. Labor hour in practical application can be divided into different types based on wage rate, the calculation method is the same.

r_i— the net return rate of fixed capital. The fixed capital rental rate $(r_i + \delta_i)$ subtracted by depreciation rate δ_i equals the net return of fixed capital.

δ_i— the depreciation rate of fixed capital. The corporate income tax levied should be based on the deduction of capital depreciation, and capital depreciation rate is usually determined by the tax authorities in accordance with the service life of fixed capital. It should be noted that the deducted capital depreciation may not be the same as the actual capital depreciation. It can also be calculated according to the actual capital depreciation rate in order to compare the cost price of the products.

w_i— the wage rate of labor hours.

C_i— the fixed cost invested, which is related with the fixed capital invested K_i, the fixed-capital rental rate $(r_i + \delta_i)$, the invested labor hours L_i, the wage rate w_i of labor hours, etc. Fixed cost calculation method can refer to Eq. (3).

h_i— the marginal cost to produce one unit of the *ith* product, a constant. For instance, the value volume of raw materials bought from the outside of the supply chain network, etc.

τ— the rate of VAT, which is the same among different production units. Each production unit also consumes public fixed capital and public service apart from consumables or intermediate input such as raw materials, fixed capital input and labor hour input. The difference between output and intermediate input is called the value added of the production unit, and the value-added tax levied on the value added is used to compensate for public fixed capital and expenses public service expenses. VAT rate is normally specified by tax authorities.

σ— the rate of corporate income tax, which is the same among different production units. Corporate income or gross profit can be obtained after VAT and the deduction of fixed capital depreciation and salary. Corporate income tax is levied to compensate for public fixed capital and public service expenses. Corporate income tax rate is normally specified by tax authorities.

q_i— the production quantity of the *ith* product, which can be the quantity of the products, or the ton-kilometers of transportation accomplished by transportation departments, etc.

p_i— the price of the *ith* product. It should be noted that the products produced by one production unit may be the input of another production unit. The two production units belong to one independent financial accounting department but without market transaction. Thus, the price here is not the market price but the production cost of product produced by production unit *i*.

a_{ji}— the quantity of the *jth* product inside the supply chain network needed to invest to produce one unit of the *ith* product.

d_i— the demand for the *ith* product from outside the network. It is the function of the product price.

During the production process, the enterprise has to use public infrastructure and public services. Therefore, the fiscal and taxation authorities levy tax on the enterprise. At present, Chinese financial and tax authorities mainly collect value-added tax and corporate income tax (and sometimes other additional small taxes). After the tax rates are enacted and implemented for a period of time, if the companies complain of the heavy tax burden, the government needs to reduce the tax rates. If there is a financial deficit (revenue less than expenditure), the government will consider increasing the tax rate. Tax rate is not adjusted arbitrarily because it is fixed over a long period.

Capital depreciation needs to be deducted when the government levies corporate income tax, and the rate of capital depreciation is determined. In China, according to 《Approach for the Pre-tax Deduction of Corporate Income Tax》 issued by State Administration of Taxation, the capital depreciation rate is determined according to the service life of different types of fixed assets (plant, equipment, etc.). It should be noted that the deducted capital depreciation may not be the same as the actual capital depreciation. It can also be calculated according to the actual capital depreciation rate in order to compare the cost price of the products.

First, the equilibrium of output quantity is taken into account.

As the quantity of fixed capital invested is a given constant which is measured in monetary unit, only the equilibrium of the quantity material objects inside the supply chain network is needed to be taken into account. The outputs of various products should equal the corresponding demands of the production activities of each unit both inside and outside the supply chain network. Thus, the following equilibrium equation is obtained.

$$q_i = a_{i1}q_1 + \cdots + a_{in}q_n + d_i. \tag{4}$$

The matrix form of this equation is expressed as follows.

$$q = Aq + d(p), \tag{5}$$

where

$$q = [q_1, \ldots, q_n]^{\tau},\ p = [p_1, \ldots, p_n],\ d(p) = [d_1, \ldots, d_n]^{\tau},\ A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}.$$

Eq. (5) is consistent with Leontief static input–output model in forms.

The output level of production activity of each unit can be calculated according to Eq. (5):

$$q = (I - A)^{-1} \times d(p). \tag{6}$$

It can be seen from Eq. (6) that the gross output of the *ith* product is the function of price.

$$q_i = q_i(p). \tag{7}$$

Now the equilibrium between income and cost of each unit is taken into account.

If the output of the *ith* product is q_i, then the income is p_iq_i. The value-added equals the sales income minus the intermediate input,i.e.

The Value-added $= p_iq_i - p_1a_{1i}q_i - \ldots - p_na_{ni}q_i - h_iq_i$

If the rate of VAT is τ, then VAT to be turned over is as follows:

$$\text{The VAT to be turned over} = \tau(p_iq_i - p_1a_{1i}q_i - \cdots - p_na_{ni}q_i - h_iq_i). \tag{8}$$

The income of enterprises after turning over the VAT is as follows:

The income of enterprises after turning over the VAT $= (1-\tau)(p_iq_i - p_1a_{1i}q_i - \cdots - p_na_{ni}q_i - h_iq_i)$

The tax base is obtained by subtracting the fixed capital depreciation and the wage from the corporate income after the VAT is turned over, and it is multiplied with the corporate income tax rate to acquire corporate income tax needed to be turned over. Consequently, the following can be obtained:

After the deduction of the fixed capital depreciation and the wages, the enterprise income is as follows:

The enterprise income $= (1-\tau)(p_iq_i - p_1a_{1i}q_i - \cdots - p_na_{ni}q_i - h_iq_i) - \delta_iK_i - w_iL_i$.

The tax base = The enterprise income

The corporate income tax needed to be turned over

$$= \sigma[(1-\tau)(p_iq_i - p_1a_{1i}q_i - \cdots - p_na_{ni}q_i - h_iq_i) - \delta_iK_i - w_iL_i].$$

Then, the tax amount T_i turned over by the *ith* unit equals the sum of the VAT and the corporate income tax, i.e.

$$T_i = \sigma[(1-\tau)(p_iq_i - p_1a_{1i}q_i - \cdots - p_na_{ni}q_i - h_iq_i) - \delta_iK_i - w_iL_i] + \tau(p_iq_i - p_1a_{1i}q_i - \cdots - p_na_{ni}q_i - h_iq_i). \tag{9}$$

The cost pricing equation can be expressed with the following equation:

$$\text{Income} = \text{cost}. \tag{10}$$

The cost consists of the following items: the VAT turned over, the corporate income tax, the fixed cost used C_i, the marginal cost h_iq_i incurred by purchasing raw materials from outside the network, as well as the marginal cost incurred by purchasing the products inside the supply chain network, i.e.

$$\text{Income} = \text{the VAT turned over} + \text{the corporate income tax} + \text{the fixed cost used} + \text{the marginal cost}. \tag{11}$$

Since the quantity of various products inside the supply chain network invested to produce one unit of the ith product is $a_{1i}, \ldots, a_{ni}$, the corresponding marginal cost of the output of q_i is $p_1a_{1i}q_i + \cdots + p_na_{ni}q_i$. Accordingly, the equation where the income equals the cost is as follows:

$$p_iq_i = T_i + p_1a_{1i}q_i + \cdots + p_na_{ni}q_i + (r_i + \delta_i)K_i + w_iL_i + h_iq_i. \tag{12}$$

By substituting Eq. (9) into the above equation, we have

$$\begin{aligned} p_iq_i = {} & \sigma[(1-\tau)(p_iq_i - p_1a_{1i}q_i - \cdots - p_na_{ni}q_i - h_iq_i) - \delta_iK_i - w_iL_i] + \tau(p_iq_i - p_1a_{1i}q_i - \cdots - p_na_{ni}q_i - h_iq_i) + p_1a_{1i}q_i + \cdots \\ & + p_na_{ni}q_i + (r_i + \delta_i)K_i + w_iL_i + h_iq_i, \end{aligned}$$

which can be changed into

$$\begin{aligned} p_iq_i = {} & [\sigma(1-\tau) + \tau]p_iq_i + (1-\sigma)(1-\tau)(p_1a_{1i}q_i + \cdots + p_na_{ni}q_i) + (1-\sigma)(1-\tau)h_iq_i + (1-\sigma)[(r_i + \delta_i)K_i + w_iL_i] \\ & + \sigma r_iK_i. \end{aligned} \tag{13}$$

Divide both sides of the equation by q_i, then the cost-based pricing equation can be obtained as follows:

$$\begin{aligned} p_i = {} & [\sigma(1-\tau) + \tau]p_i + (1-\sigma)(1-\tau)(p_1a_{1i} + \cdots + p_na_{ni}) + (1-\sigma)(1-\tau)h_i + \{(1-\sigma)[(r_i + \delta_i)K_i + w_iL_i] \\ & + \sigma r_iK_i\}/q_i(p). \end{aligned} \tag{14}$$

3. The existence and uniqueness of solution to cost-based pricing of various products in a short term

On the right of Eq. (14) is the average cost for producing one unit of product of the *ith* unit. Therefore, the average cost function $\Phi_i(p)$ of the *ith* unit can be defined as follows:

$$\begin{aligned} \Phi_i(p) = {} & [\sigma(1-\tau) + \tau]p_i + (1-\sigma)(1-\tau)(p_1a_{1i} + \cdots + p_na_{ni}) + (1-\sigma)(1-\tau)h_i + \{(1-\sigma)[(r_i + \delta_i)K_i + w_iL_i] \\ & + \sigma r_iK_i\}/q_i(p). \end{aligned} \tag{15}$$

The average cost vector $\Phi(p)$ can be rewritten as follows:

$$\Phi(p) = [\Phi_1(p), \ldots, \Phi_n(p)]. \tag{16}$$

The income from the production of one unit of product by unit i is p_i; thus the income vector can be designated as follows:

$$p = [p_1, \ldots, p_n]. \tag{17}$$

According to Eqs. (14) and (15), the cost-based pricing equation can be defined as follows, and it can be taken as the mapping in n dimensional space.

$$p = \Phi(p). \tag{18}$$

In order to discuss the existence and uniqueness of the solution to the above cost-based pricing equation, it is supposed that the following conditions are satisfied:

Condition 1: The demand function is the monotone decreasing function of the price, e.g. if the price p and s are given, and $p > s > 0$, then $d(s) > d(p)$.

It is worth pointing out that the demand equation $d(s)$ does not necessarily satisfy the condition mentioned above in some cases. It will be seen from the following discussion that if the demand function satisfies this condition, iteration algorithm can be used to solve Eq. (18), and the existence and uniqueness of the solution to Eq. (18) can be easily to solve. If the demand function $d(s)$ does not satisfy Condition 1, then the discussions on the existence uniqueness of the solution to Eq. (18) may be more complex, and this situation is not investigated in this study.

Condition 2: If $p \to 0$, then $d(p) \to \bar{d}$ and $\bar{d} = [\bar{d}_1, \ldots, \bar{d}_n]$. If $p \to \bar{p}$ and $\bar{p} = [\bar{p}_1, \ldots, \bar{p}_n]$, then $d(p) \to 0$. Where, $\bar{d}$ and $\bar{p}$ are the given constant vectors.

This condition suggests that if the price tends towards 0, then the demand quantity tends towards a relatively larger positive constant vector. If the price tends towards a certain larger positive constant vector, then the demand quantity tends towards 0.

Condition 3: There exists $p = p^{\otimes} > 0$, where the income is larger than the cost, i.e. $p^{\otimes} > \Phi(p^{\otimes})$.

This condition indicates that when the market price $p^{\otimes} > 0$, the income $p^{\otimes}$ is larger than the cost $\Phi(p^{\otimes})$, which implies that the production activity is profitable.

Condition 4: There exists $p = p^{\Delta} > p^{\otimes} > 0$, where the income is less than the cost, i.e. $p^{\Delta} < \Phi(p^{\Delta})$.

This condition suggests that when the market price is too high, the demand quantity tends towards 0, and the production activity is losing money.

The discussion of the existence and uniqueness of the solution to cost-based pricing Eqs. (18) or (14) requires the following lemma 1, which can be found in Ortega and Rheinboldt [17].

Lemma 1 (*Kantorovich*). *Consider following nonlinear equation:*

$$p = \Omega(p). \tag{19}$$

The $\Omega(p)$ is the one to one map on D: $D \subset R^n \to R^n$.

The $\langle \eta, \xi \rangle$ denotes the set: $\{z \in R^n | \eta \leqslant z \leqslant \xi\}$, and suppose that $\langle \eta, \xi \rangle \subset D$. If the $\Omega(p)$ satisfies following conditions:

- $\Omega(\eta) > \eta$;
- If $z \geqslant g \geqslant 0$, then $\Omega(z) \geqslant \Omega(g)$, this means that $\Omega(p)$ is an isotonic map;
- There is $\xi > 0$, so that $\Omega(\xi) < \xi$.

Then the following sequence:

$$p^{k+1} = \Omega(p^k), \quad p^0 = \eta,$$

will converge to solution p^*, such that:

$$p^* = \Omega(p^*),$$

the following sequence:

$$p^{k+1} = \Omega(p^k), \quad p^0 = \xi,$$

will converge to another solution p^{**}, such that:

$$p^{**} = \Omega(p^{**}).$$

The relation between p^* and p^{**} is $p^* \leqslant p^{**}$.

The existence of the solution to cost-based pricing Eqs. (18), (14) can be described according to Theorem 1.

Theorem 1. *In Eqs. (18) or (14), if conditions 1–3 are met for this equation, then there exists a meaningful solution p^* such that*

$$p^* = \Phi(p^*). \tag{20}$$

Proof. Firstly, it can be derived from Eq. (15) that

$$\begin{aligned}\Phi_i(0) =& [\sigma(1-\tau)+\tau] \times 0 + (1-\sigma)(1-\tau)(0 \times a_{1i} + \cdots + 0 \times a_{ni}) + (1-\sigma)(1-\tau)h_i \\ &+ \{(1-\sigma)[(r_i+\delta_i)K_i + w_iL_i] + \sigma r_i K_i\}/q_i(0) > 0,\end{aligned}$$

which means

$$\Phi(0) > 0. \tag{21}$$

Secondly, because the demand is a decreasing monophonic function of price, this means that $\Phi(p)$ is an increasing monophonic function of p. That is: if $p > s > 0$, then $\Phi(p) > \Phi(s)$.

Thirdly, according to condition 3, there exists $p = p^{\otimes} > 0$, which makes $p^{\otimes} > \Phi(p^{\otimes})$.

According to lemma 1, the following sequence is tenable:

$$p^{k+1} = \Phi(p^k), \quad p^0 = 0,$$

which converges to solution p^* such that:

$$p^* = \Phi(p^*).$$

The following sequence is tenable

$$p^{k+1} = \Phi(p^k), \quad p^0 = p^{\otimes},$$

which converges to another solution p^{**} such that:

$$p^{**} = \Phi(p^{**}).$$

The relation between p^* and p^{**} is that $p^* \leqslant p^{**}$. $\square$

The uniqueness of the solution to cost-based pricing Eqs. (18), (14) is discussed as follows.

When price $p = 0$, then $0 < \Phi(0)$, which indicates that the supply chain network is losing money. According to Condition 3, when the price rises so that $p = p^{\otimes} > 0$, then the income is larger than the cost, i.e. $p^{\otimes} > \Phi(p^{\otimes})$. When the supply chain network is making profits, then according to Theorem 1, there exists at least 1 breakeven point between $p = 0$ and $p = p^{\otimes} > 0$, or, there exists the cost-based price and the output. When the price keeps soaring, then according to Condition 2, if $p \to \bar{p}$, then $d(\bar{p}) \to 0$, where $\bar{p}$ is the given constant vector. According to Eq. (15), $d(\bar{p}) \to 0$, which indicates that the average cost $\Phi(\bar{p}) \to \infty$. Since $\bar{p}$ is a limited constant vector, then $\bar{p} < \Phi(\bar{p})$ can also be obtained, indicating that the supply chain network is losing money. Consequently, there also exists at least one breakeven point between $p = p^{\otimes} > 0$ and $p = \bar{p}$, or, there exist cost-based price and the output. Therefore, if the conditions 1–4 are all satisfied, then there usually exist at least two different breakeven points in supply chain network, or, the solutions to the cost-based pricing equation may be not unique.

It is worth noticing that the Cournot equilibrium solution was frequently discussed in the supply chain network. Cournot equilibrium solution usually exists and it is unique, whereas the equilibrium solution of price also exists and it is unique in the general long-term equilibrium pricing model (see [11,18]). The existing literatures often focus on the uniqueness of equilibrium price. However, this paper demonstrates that there are usually many equilibrium price solutions in a supply chain network. In the following example, a supply chain network involving seven business units is given with four equilibrium price solutions. The first one is the lowest equilibrium price solution and it is also the most important solution, since it reflects the fair pricing of products. Its stable balanced point may be found out by the iteration method according to Theorem 1. There may exist the highest equilibrium price solution. It is the unstable balanced point, which cannot be found directly according to iteration method. The prices obtained with other equilibrium price solutions range from the highest to the lowest. The existence, uniqueness and computability of the solution to mathematical economic model are the fundamental problems that must be solved. This paper reveals that the solution to cost-based pricing is not unique and the iterative algorithm is proposed to obtain the lowest cost price solution. But some problems require further research:

A. When the node of the supply chain network is relatively larger, there may be multiple solutions to cost-based pricing models simultaneously. How many are the solutions to the cost-based pricing model?
B. How many are stable equilibrium points in solutions to the cost-based pricing model? Is the lowest solution to cost-based pricing model the only stable equilibrium point?
C. How to calculate all the solutions to cost-based pricing model for a large system?
D. Is the profitable area of the supply chain network the area confined by these solutions? Intuitively speaking, it should be like that but it needs strict proof.

We now come to analyze the influence of tax on supply chain network.

Theorem 2 deals with the relationship between the tax increase and the value-added of supply chain network.

Theorem 2. *In (18) or (14), if conditions 1–3 are met for this equation, then the tax revenue equals the increase in the value-added of supply chain network after tax.*

Proof. Suppose that before tax, the equilibrium price is p_i°, and output level is q_i°, while after tax, the equilibrium price is p_i^*, and the output level is q_i^* according to Eq. (12), then the balanced equation after tax is

$$p_i^* q_i^* = T_i + p_1^* a_{1i} q_i^* + \cdots + p_n^* a_{ni} q_i^* + (r_i + \delta_i) K_i + w_i L_i + h_i q_i^*.$$

The value-added after tax is

$$p_i^* q_i^* - (p_1^* a_{1i} q_i^* + \cdots + p_n^* a_{ni} q_i^*) - h_i q_i^* = T_i + (r_i + \delta_i) K_i + w_i L_i.$$

The value-added before tax is

$$p_i^\circ q_i^\circ - (p_1^\circ a_{1i} q_i^\circ + \cdots + p_n^\circ a_{ni} q_i^\circ) - h_i q_i^\circ = (r_i + \delta_i) K_i + w_i L_i.$$

The increase in the value-added after tax is

$$[p_i^* q_i^* - (p_1^* a_{1i} q_i^* + \cdots + p_n^* a_{ni} q_i^*) - h_i q_i^*] - [p_i^\circ q_i^\circ - (p_1^\circ a_{1i} q_i^\circ + \cdots + p_n^\circ a_{ni} q_i^\circ) - h_i q_i^\circ] = T_i \qquad \square.$$

Theorem 2 indicates that the taxation will increase the value-added, which seems to mean that taxation is beneficial. However, Theorem 3 shows that the taxation will reduce the consumer surplus, which subjects consumers to certain losses.

Theorem 3. *In Eqs. (18) or (14), if conditions 1–3 are met for this equation, then total tax revenue is less than the tax burden on consumers, which is defined as the increase in consumer spending after tax.*

Proof. Suppose that before tax, the equilibrium price is p_i°, and output level is q_i°, while after tax, the equilibrium price is p_i*, and output level is q_i*, then the tax burden on consumers .is as follows

$$\text{The tax burden on consumers in unit } i = (p_i^* - p_i^\circ)q_i^*. \tag{22}$$

By Eq. (12), the balanced equation after tax can be changed to

$$p_i^* = T_i/q_i^* + (p_1^* a_{1i} + \cdots + p_n^* a_{ni}) + [(r_i + \delta_i)K_i + w_i L_i]/q_i^* + h_i. \tag{23}$$

By Eq. (12), the balanced equation before tax can be changed to

$$p_i^\circ = (p_1^\circ a_{1i} + \cdots + p_n^\circ a_{ni}) + h_i + [(r_i + \delta_i)K_i + w_i L_i]/q_i^\circ. \tag{24}$$

By substituting Eqs. (23) and (24) into Eq. (22), we have the following equation:

$$\text{The tax burden on consumers in unit } i = (p_i^* - p_i^\circ)q_i^*,$$

$$= T_i + [(p_1^* - p_1^\circ)a_{1i} + \cdots + (p_n^* - p_n^\circ)a_{ni}]q_i^* + [(r_i + \delta_i)K_i + w_i L_i][1 - q_i^*/q_i^\circ] > 0,$$

which means

$$T_i < \text{The tax burden on consumers in unit } \; i,$$

or

$$\sum T_i < \text{The tax burden on consumers} \qquad \square. \tag{25}$$

4. An example

In actual production activities, the process from raw materials to semi-finished products and finally to the output of finished products consists of many links, each of which can be seen as an independent production unit. All production units are interconnected so as to form a supply chain network. Each production unit produces one only kind of product (it can be finished product, and also can be semi-finished products as the intermediate input of the next production unit). To produce this kind of product, 3 types of production factors shall be input: A. Intermediate inputs, such as raw materials, energy consumption and to be semi-finished products from the preceding product unit. B. Fixed capital investment such as plant, equipment. C. Labor hours: managers, technicians, workers. Cost calculation, management and control is a must for any modern enterprise. Therefore, the above data must be available and accessible. But the production activities in real life usually consist of many production units, and therefore data acquisition and analysis takes much time. Due to limited space, only simple calculation examples are given in this paper. The given example consists of 7 production units, which is simplification of real production activities (each production unit has only one type of fixed capital). It is not a real example from a real production background. In addition, it should be noted that an enterprise is divided into a number of production units, and the sum of value added of all production units is the total value added of the enterprise. Therefore, levying VAT on the enterprise has the same effect as on every production unit. The same situation applies to corporate income tax

Fig. 1 shows the simple production and business system consisting of seven units. The products from the first and the second manufacturing units should be transported via the forth unit to the third unit for assembly. One part of the assembled products is transported to the fifth unit for sales, while the other part is transported to the seventh manufacturing unit, and they are assembled with the products from the sixth manufacturing unit. The assembled products are directly for sales.

In practical applications, we must first consider a firm or company with independent accounting, which can be divided into several production units. They are interconnected with each other to form the supply chain network. It should be noted that the Production unit 4 belongs to the transport department of the company. In reality, there exist logistics companies with independent economic accounting that do not belong to the production company. For example, Production unit 1 purchases raw material from the outside, and then an external logistics company transports the material to the company. In this case, we only need to include the transportation cost of delivering the raw material (including the delivery expense paid to the logistics company) from the outside to the Production unit 1 into the marginal cost h_1.

However, the real situation is more complex. For example, a production unit processes a kind of product at this time, and at other time it processes another kind of product. In this case, this unit can be subdivided into two smaller production units sharing the fixed capital and labor hours according to the processing time. The concrete solution is determined in accordance with the actual situation. The example cited in the paper is the simplification of the actual situation.

Parameters of matrix A are given in Eq. (26). The parameters in the third column of the matrix are $a_{13} = 1.05$ and $a_{23} = 1.03$, respectively, implying that 1.05 units of products from the first unit and 1.03 units of products from the second unit is needed to be invested to produce one unit of product of the third unit. The reason for 1.05 > 1 and 1.03 > 1 lies in that there exists certain rejection rate during the manufacturing process. If the rejected products do not appear, then one unit of product of the third unit can be produced by assembling one unit of product from unit one and one unit of product from unit two.

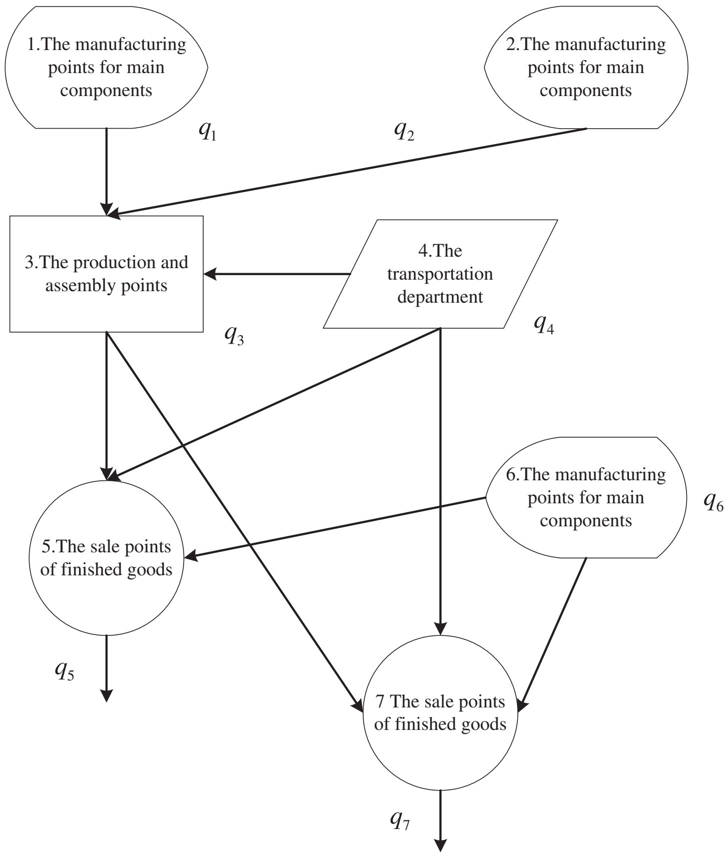

Fig. 1. The production flow chart of a corporation.

We note that a_{ji} is the quantity of the *jth* product inside the supply chain network needed to invest to produce one unit of the *ith* product.

a_{35} = 1.01 means that 1.01 units of product 3 are required to produce one unit of product 5. a_{45} = 3.1 means that 3.1 units of product 4 are required to produce one unit of product 5. (For example, it is required to transport 1.01 units of product 3 from the Production unit 3 to the Production unit 5 in order to produce one unit of product 5. The weight of 1.01 units of product 3 is a ton, while the distance between the Production unit 3 and 5 is b kilometer, so that $a \times b = 3.1$ ton-kilometer. Each ton-kilometer is one unit of output of Production unit 4, which means that 3.1 units of Product 4 are required to produce one unit of Product 5.) a_{27} = 0 means that product 2 is not required to produce product 7. a_{37} = 1.02 and a_{67} = 1.1 mean that 1.02 units of product 3 and 1.1 units of product 6 are required to produce one unit of product 7. A total of 2.5 units of output of Production unit 4 are required to transport 1.02 units of product 3 and 1.1 units of product 6 to Production unit 7. This means a_{47} = 2.5, that is, 2.5 units of product 4 are required to produce one unit of product 7.

$$A = \begin{bmatrix} a_{11} & \cdots & a_{17} \\ \vdots & & \vdots \\ a_{71} & \cdots & a_{77} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1.05 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.03 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.01 & 0 & 1.02 \\ 0 & 0 & 2.20 & 0 & 3.1 & 0 & 2.5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$(I-A)^{-1} = \begin{bmatrix} 1 & 0 & 1.05 & 0 & 1.0605 & 0 & 1.071 \\ 0 & 1 & 1.03 & 0 & 1.0403 & 0 & 1.0506 \\ 0 & 0 & 1 & 0 & 1.01 & 0 & 1.02 \\ 0 & 0 & 2.2 & 1 & 5.322 & 0 & 4.744 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

According to Eq. (7), we have (it should be noted that $d_1 = d_2 = d_3 = d_4 = d_6 = 0$):

$$q = (I-A)^{-1}d = \begin{bmatrix} 1 & 0 & 1.05 & 0 & 1.0605 & 0 & 1.071 \\ 0 & 1 & 1.03 & 0 & 1.0403 & 0 & 1.0506 \\ 0 & 0 & 1 & 0 & 1.01 & 0 & 1.02 \\ 0 & 0 & 2.2 & 1 & 5.322 & 0 & 4.744 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \\ d_7 \end{bmatrix},$$

$$q_1 = 1.0605d_5 + 1.071d_7, \tag{27}$$

$$q_2 = 1.0403d_5 + 1.0506d_7, \tag{28}$$

$$q_3 = 1.01d_5 + 1.02d_7, \tag{29}$$

$$q_4 = 5.322d_5 + 4.744d_7, \tag{30}$$

$$q_5 = d_5, \tag{31}$$

$$q_6 = 1.1d_7, \tag{32}$$

$$q_7 = d_7. \tag{33}$$

Suppose the final demands of Product 5 and 7 are as follows:

$$d_5 = 95(2.5 - 0.9p_5), \tag{34}$$

$$d_7 = 190 - 65p_7. \tag{35}$$

The data of the fixed capital in each of workshop and the marginal cost for producing unit product are shown in Table 1. The parameters of the tax rate, capital depreciation rate, etc. are as follows:

$$\sigma = 0.2, \tau = 0.15, \delta = 0.1, r = 0.05, w = 3.$$

It can be derived from Eq. (14) and Eqs. (27)–(35) that

$$p_1 = 0.32p_1 + 0.068 + 15.25/(1.0605d_5 + 1.071d_7),$$

$$p_2 = 0.32p_2 + 0.0476 + 3.05/(1.0403d_5 + 1.0506d_7),$$

$$p_3 = 0.32p_3 + 0.034 + 0.68(1.05p_1 + 1.03p_2 + 2.2p_4) + 24.4/(1.01d_5 + 1.02d_7),$$

$$p_4 = 0.32p_4 + 0.0204 + 7.98/(5.322d_5 + 4.744d_7),$$

$$p_5 = 0.32p_5 + 0.68(1.01p_3 + 3.1p_4) + 9.44/d_5,$$

Table 1
The data of the fixed capital and the marginal cost.

	1	2	3	4	5	6	7
K_i	25.0	5.0	40.0	6.0	8.0	5.5	7.0
L_i	5.0	1.0	8.0	3.0	3.5	2.8	0.6
h_i	0.1	0.07	0.05	0.03	0.0	0.04	0.0

$$p_6 = 0.32p_6 + 0.0272 + 7.435/1.1d_7,$$

$$p_7 = 0.32p_7 + 0.68(1.02p_3 + 2.5p_4 + 1.1p_6) + 2.35/d_7.$$

The lowest cost price can be obtained by the iterative method with the calculation steps as follows:
Step 1: Set the initial number of computational cycles $t = 0$.
Step 2: Set the error ε.
Step 3: Set the initial price $p^t = 0$, i.e. $p_1^t = p_2^t = \cdots = p_7^t = 0$.
Step 4: Calculate the demand of the tth cycle according Eqs. (34) and (35):

$$d_5^t = 95(2.5 - 0.9p_5^t),$$

$$d_7^t = 190 - 65p_7^t.$$

Step 5: Calculate cost price tth cycle according to Eq. (14) or (27)–(35);

$$p_1^{t+1} = 0.32p_1^t + 0.068 + 15.25/(1.0605d_5^t + 1.071d_7^t),$$

$$p_2^{t+1} = 0.32p_2^t + 0.0476 + 3.05/(1.0403d_5^t + 1.0506d_7^t),$$

$$p_3^{t+1} = 0.32p_3^t + 0.034 + 0.68(1.05p_1^t + 1.03p_2^t + 2.2p_4^t) + 24.4/(1.01d_5^t + 1.02d_7^t),$$

$$p_4^{t+1} = 0.32p_4^t + 0.0204 + 7.98/(5.322d_5^t + 4.744d_7^t),$$

$$p_5^{t+1} = 0.32p_5^t + 0.68(1.01p_3^t + 3.1p_4^t) + 9.44/d_5^t,$$

$$p_6^{t+1} = 0.32p_6^t + 0.0272 + 7.435/1.1d_7^t,$$

$$p_7^{t+1} = 0.32p_7^t + 0.68(1.02p_3^t + 2.5p_4^t + 1.1p_6^t) + 2.35/d_7^t.$$

Step 6: If the error is less than ε, i.e. $|p_j^{t+1} - p_j^t| \leqslant \varepsilon, j = 1, \cdots, 7$, then the calculation is terminated at step 8.
Step 7: Set $t = t + 1$ and go back to step 4 and carry out calculation of the next cycle.
Step 8: Output the calculation results when the calculation is over.

Here the conditions 1–3 are satisfied, so there exit many solutions for the above equations. The calculation results of the prices and the output levels are shown in Tables 2–5.

Fig. 2 shows the relations among the four equilibrium price solutions. Fig. 3 shows the relations among the four equilibrium output level solutions.

Where p^* is the lowest equilibrium price solution, $p^\wedge$ is the highest equilibrium price solution, $p_i^\exists$ and p_i^∇ are the middle equilibrium price solutions ranging from the highest price to the lowest.

Table 2
The first group of solutions corresponding to the lowest price level (based on the distance from the price vectors to the original).

	1	2	3	4	5	6	7
p_i^*	0.166649	0.08358862	0.5051833	0.03733427	0.7252858	0.1178695	0.7628916
q_i^*	339.2874	332.8247	323.1308	1613.7	177.4588	155.1835	141.0759

Table 3
The second group of solutions corresponding to the highest price level (based on the distance from the price vectors to the original).

	1	2	3	4	5	6	7
$p_i^\wedge$	0.52370	0.15638	1.64098	0.07646	2.43820	0.52083	2.60504
$q_i^\wedge$	52.93059	51.92239	50.41009	252.5883	29.0391	22.73964	20.6724

Table 4
The third group of solutions corresponding to the lower price level (based on the distance from the price vectors to the original).

	1	2	3	4	5	6	7
$p_i^\exists$	0.26874	0.10440	0.83265	0.04976	2.67943	0.12586	1.14200
$q_i^\exists$	132.9071	130.3756	126.5782	593.9642	8.40874	127.347	115.77

Table 5
The fourth group of solutions corresponding to the higher price level (based on the distance from the price vectors to the original).

	1	2	3	4	5	6	7
p_i^{∇}	0.23022	0.09655	0.70598	0.04365	0.96623	1.37694	2.80870
q_i^{∇}	172.2204	168.94	164.0194	859.5797	154.8873	8.177944	7.434494

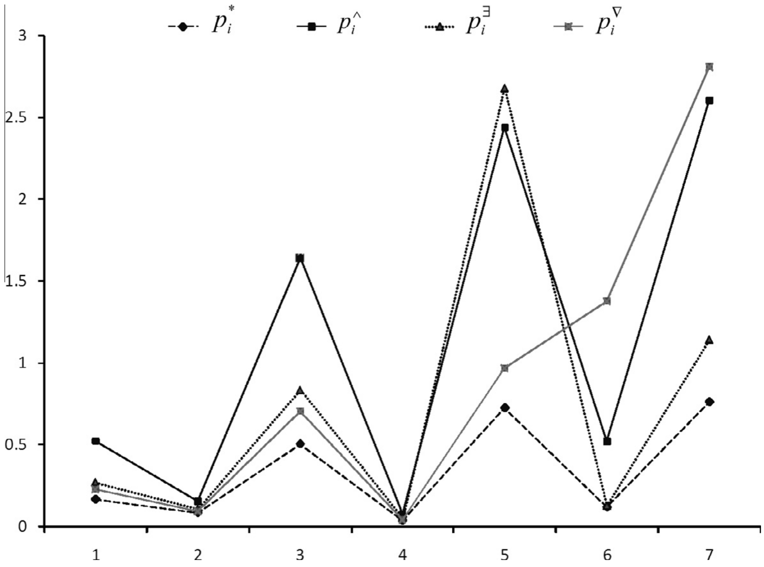

Fig. 2. The 4 equilibrium price solutions.

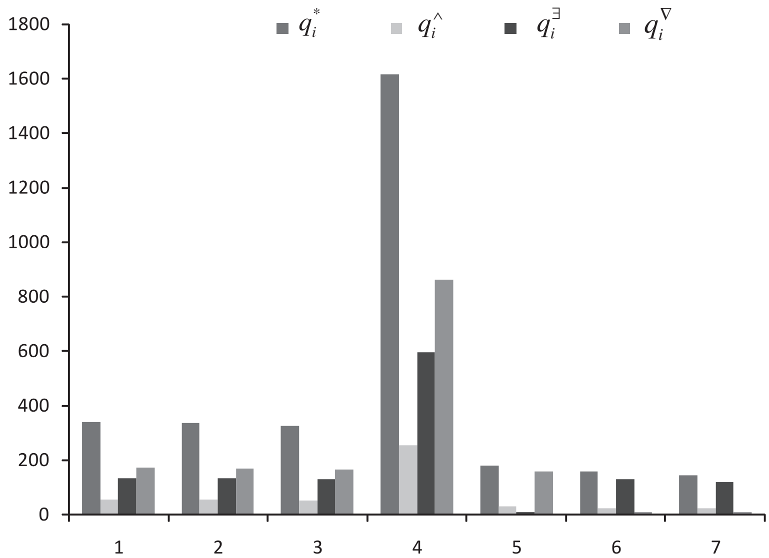

Fig. 3. The 4 equilibrium output level solutions.

In the case of the lowest equilibrium price solution, the value-added before tax is 86.174; the value-added after tax is 102.801; the total tax revenue is 16.626; the total revenue equals the increase in the value-added. These calculation results conform to Theorem 2.

When tax is taken into account, the expenditure for purchasing all products is

$$\sum p_i^* q_i^* = 554.902,$$

where p_i° is the equilibrium price without considering tax, the expenditure for purchasing all products after tax is

$$\sum p_i^{\circ} q_i^* = 514.17.$$

The tax burden on consumers is 554.902 − 514.17 = 40.73. So we have

$$\sum T_i < \text{The burden on consumers}.$$

The calculation results conform to Theorem 3.

It should be pointed out that here we don't consider the maximum production capacities of every workshop, which goes beyond the scope of this paper.

5. Conclusions and recommendations for future work

The short-term cost-based pricing model of supply chain network containing n production units is built in this study. The function of average cost of each production unit decreases with the increase of output. If the demand function satisfies a certain conditions, the cost function is the monotone mapping in n dimensional space. According to Kantorovich theorem, the existence uniqueness of the equilibrium point solution to the cost-based pricing function with the VAT and corporate income tax considered has been discussed. Generally, many equilibrium solutions to the cost-based pricing function may exist. When the prices are inside the region encircled by various equilibrium points, the excess profits higher than the costs are obtained. The iteration algorithm for the equilibrium is proposed in this study, and it is illustrated with an example.

The cost-based pricing model presented in this paper can be applied to cost control and management in an enterprise or a company. There are still many other issues worthy of further research. Such as the problems in model construction: if more complex assumptions are made for cost function and demand function, then it will be more difficult to solve the cost-based pricing model. Problems in calculating model solutions are as follows: the existence, uniqueness and stability of the solution to cost-based pricing model, number of the solution to cost-based pricing model and its relationship to the profitable areas of the supply chain network. All the problems above are worthy of profound discussion. The problems in actual application are as follows: when there are many nodes or production units or if other tax categories are levied besides VAT and corporate income tax, the problem of heavy workload in model constructing will arise. Once a mathematical economic model is successfully applied in practice, it will always produce huge economic benefits.

Acknowledgments

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